

When Lempel-Ziv-Welch Meets Machine Learning: A Case Study of Accelerating Machine Learning using Coding

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Abstract

In this paper we study the use of coding techniques to accelerate machine learning (ML). Coding techniques, such as prefix codes, have been extensively studied and used to accelerate low-level data processing primitives such as scans in a relational database system. However, there is little work on how to exploit them to accelerate ML algorithms. In fact, applying coding techniques for faster ML faces a unique challenge: one needs to consider both how the codes fit into the *optimization algorithm used to train a model*, and the interplay *between the model structure and the coding scheme*. Surprisingly and intriguingly, our study demonstrates that a slight variant of the classical Lempel-Ziv-Welch (LZW) coding scheme is a good fit for several popular ML algorithms, resulting in substantial runtime savings. Comprehensive experiments on several real-world datasets show that our LZW-based ML algorithms exhibit speedups of up to 31x compared to a popular and state-of-the-art ML library, with no changes to ML accuracy, even though the implementations of our LZW variants are not heavily tuned. Thus, our study reveals a new avenue for accelerating ML algorithms using coding techniques and we hope this opens up a new direction for more research.

1 Introduction

Machine learning (ML) is a critical component for numerous data-driven applications [12, 13, 19, 20, 23]. Naturally, there is a lot of interest in techniques that could accelerate the runtime performance of ML algorithms. As part of this broad direction, a recent line of research has focused on closer integration of ML and data processing techniques [12, 13, 20], as well as exploiting data processing techniques to accelerate ML algorithms [23].

The focus of this paper is to explore *the use of coding techniques to accelerate ML algorithms*. Coding techniques, such as prefix codes, have been extensively applied to accelerate low-level data processing primitives, such as scans in relational database systems [24], and more recently, primitive linear algebra operations [11]. Typically, for a target operation, these prior works find an appropriate coding scheme for the input data and create algorithms to *directly work on the codewords*. This typically helps avoid *redundant computations*, thus improving runtime performance. Carefully synthesizing the coding scheme and the algorithm that works on the codewords often yields substantial runtime speedups. It is thus natural, and indeed tantalizing, to examine whether similar ideas can be applied to ML algorithms, which can be viewed as a more advanced data processing methods.

Despite its naturalness, there has been little work on studying how to accelerate ML algorithms using coding techniques. In fact, the lack of understanding of this approach does not seem to be a coincidence: a

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closer examination of this idea reveals some unique challenges in applying coding techniques to accelerate ML algorithms. To this end, we note that for a successful application of this idea, one has to carefully consider the interplay between the coding scheme and the computation. For low-level primitives, this interplay is relatively easy, since the computations are elementary (consider, for example, scans or matrix multiplications) and the patterns can be easily observed and exploited.

However, ML algorithms typically perform *a much more complicated computation over the data*. In fact, intuitively, we observe that there are three aspects that should be considered for ML over encoded data: (1) the *structure of the ML model* we want to learn (with its associated *loss function*), (2) the *optimization algorithm* (for example, batch gradient descent or stochastic gradient descent), and (3) the *coding scheme*, which creates opportunities for new techniques. A successful approach must simultaneously consider the interplay of all three aspects—this might become significantly more complicated than just the interplay between the coding scheme and the algorithm in the case of low-level data processing primitives.

In this paper, we take a first step towards examining the interplay of all three aspects. Surprisingly and intriguingly, we find that a slight variant of the classical Lempel-Ziv-Welch (LZW) coding scheme fits well for two popular ML techniques: The *k-means clustering* and *generalized linear models* (GLMs). The essence of our idea is to exploit the *data redundancy* found by the LZW algorithm, encoded as a *prefix tree*, in both (1) “*caching intermediate results during model optimization*”, and (2) *efficient computation of the “cache” for (1)*. By combining these two opportunities with some care in choosing the codewords, we show that substantial speedups are possible for these two important ML tasks. Specifically, this paper makes the following contributions.

- We propose a set of techniques to modify the LZW algorithm, collectively called “Tuple Oriented Coding” (TOC), so as to tailor LZW to the task of accelerating ML algorithms. In particular, while traditional LZW treats the entire dataset as a large blob of text, we need to modify it so that it becomes aware of the boundary between different data points, i.e., tuples.
- We propose and study two opportunities to exploit the data redundancy found by our modified LZW algorithm, encoded as a *prefix tree*, to accelerate the ML algorithms: (1) Based on the prefix tree, we cache intermediate results during model optimization so that redundant computations can be avoided by examining the model structure, as well as how the optimization algorithm “optimizes” the model (for example, the evaluation of the loss function), and (2) We compute the cache used in (1) efficiently by exploiting the tree structure of the prefix tree, as well as the structure of the intermediate results we use.
- We perform a comprehensive empirical study using real-world datasets to compare the runtime performance of our approach against the popular state-of-the-art ML library, `scikit-learn`. Our approach yields speedups of up to 31x on the real-world datasets. Note that our approach does not alter ML accuracy, since the LZW algorithm is lossless.

In summary, our work presents an interesting case study of how coding techniques, when appropriately synthesized, can substantially accelerate ML algorithms by reducing redundancies in not just the data but also the ML computations. We hope that our work spurs more research in this promising direction to improve the runtime performance of ML algorithms.

Outline. The rest of this paper is organized as follows. Section 2 presents the background, while Section 3 gives an overview of our approach. Section 4 explains our TOC approach, while Section 5 presents our TOC-based ML algorithms. Section 6 presents the experiments. Section 7 discusses the related work and we conclude in Section 8.

2 Preliminaries and Background

This section reviews important concepts and definitions in machine learning and source coding.

Machine Learning in the Generalized Setting. We begin with a description of machine learning in the generalized setting. Our formulation follows the work of Shalev-Shwartz et al. [32]. We have a sample space

Z from which we sample an ordered training set (z_1, \dots, z_m) . Let $\mathcal{W} \subseteq \mathbb{R}^d$ be an Hilbert space equipped with an inner product $\langle \cdot, \cdot \rangle$ and a norm $\| \cdot \|$. We are given a loss function $\ell : \mathcal{W} \times Z \mapsto \mathbb{R}$. Our goal is to minimize the *empirical risk* over the training set S (i.e., the empirical risk minimization, or ERM), defined as

$$L_S(w) = \frac{1}{m} \sum_{i=1}^m \ell(w, z_i). \quad (1)$$

For fixed S , we think of $\ell_i(w) = \ell(w, z_i)$ as a function of w , also called as loss function. As pointed out in [32], this formulation encompasses many common machine learning problems. We mention two of them that will be discussed in this paper.

k-means Clustering and the Lloyd Algorithm. k -means problem is a classic machine learning problem where given a set of m points we want to partition them into k clusters so as to minimize the sum of intra-cluster distances with the centroids of the clusters. We can put k -means clustering into the framework described above. Let the sample space Z be a set of m points in \mathbb{R}^d , and the hypothesis space \mathcal{W} be all sets of k \mathbb{R}^d points. That is, a hypothesis w is a set of k points $w_1, \dots, w_k \in \mathbb{R}^d$. Now we define the loss function as

$$\ell(w, z) = \min_{1 \leq i \leq k} \|w_i - z\|^2 \quad (2)$$

Here, w represents a set of k centroids, and ℓ defines the loss of a point z to be the distance to the nearest centroid. By plugging Equation 2 into Equation 1,

$$L_S(w) = \frac{1}{m} \sum_{i=1}^m \min_{1 \leq j \leq k} \|w_j - z_i\|^2$$

Therefore, minimizing the $L_S(w)$ gives a solution to the standard k -means problem. Note that, by changing different norms, we can have different variants of k -means problem.

It is known that solving the k -means optimization problem is NP-hard [10, 27]. Therefore, there have been many algorithms that try to find a good approximation efficiently [2, 3, 15, 26]. To this end, Lloyd algorithm is a classic algorithm which finds a local optimal. Lloyd algorithm works as follows. Given a number of clusters, k , it initializes k cluster *centroids*. In each iteration, for each example, Lloyd's algorithm computes the centroid nearest to it and reassigns the example to that cluster. At the end of an iteration, every centroid is computed again as the mean of all the examples assigned to it. We repeat this for a fixed number of iterations (*Iters*) or until the centroids do not change significantly.

Generalized Linear Models. Generalized linear models are a large class of machine learning problems in the supervised setting. The sample space Z is now of the form $X \times Y$, where $X \subseteq \mathbb{R}^d$ is a feature space, and Y is either a discrete label space (classification), or a bounded set in \mathbb{R} (regression). The loss function of a generalized linear model takes the special form of

$$\ell(w; x, y) = \ell(\langle w, x \rangle, y)$$

Logistic regression, least squares linear regression, and linear support vector machine are well-known examples of generalized linear models [17]. For example, for logistic regression, $Y = \{\pm 1\}$, and

$$\ell(w; x, y) = \log(1 + \exp(-y\langle w, x \rangle)) \quad (3)$$

By plugging Equation 3 into Equation 1,

$$L_S(w) = \frac{1}{m} \sum_{i=1}^m \log(1 + \exp(-y_i \langle w, x_i \rangle))$$

Minimizing the $L_S(w)$ gives a solution to the logistic regression problem.

Training in Machine Learning. Optimization in machine learning typically has the following structure: The model structure (for example, network structure in a neural network), model parameters and the data points will jointly determine the loss of the current model on the data points. Then an optimization algorithm will update model parameters iteratively based on the loss, to try to find the optimal model parameters. For example, a common optimization algorithm is the batch gradient descent (BGD) [29]. BGD starts with an initial w_0 , and at time t , it computes the gradient of the total loss $\nabla\ell(w_t)$ and updates w_t as follows: $w_{t+1} \leftarrow w_t - \eta_t \nabla\ell(w_t)$, where η_t is a stepsize hyper-parameter.

Source Coding (Compression). Source coding is widely used to compress data [9]. It takes an input with a stream of independently and identically distributed (i.i.d.) data points and outputs a code stream [9].

LZ77 [37] and LZ78 [38] are two dictionary-based lossless encoders. LZ77 uses a sliding window to find repeated patterns in the file and LZ78 builds a prefix tree to store the dictionary explicitly. Many variants like LZW [34] and LZSS [33] are based on LZ77 and LZ78. Besides their academic influence, these variants formed the basis of several ubiquitous compression schemes: Popular archivers like RAR¹ and PKZip² use LZSS. Unix file compression utility compress³ and GIF [36] image format are based on LZW. In this paper, we use LZW as the foundation of our coding schemes.

Lempel-Ziv-Welch (LZW) Compression. LZW [34] is a dictionary-based source coding scheme that is primarily used for text and image files. The dictionary is typically implemented as a prefix tree. Encoding takes each possible input sequence of bits of a given length (for example, 12 bits) and creates an entry in the dictionary that consists of the pattern itself and a *code*, identified by the *code number*. As the input file is scanned, the longest match with a previously encoded pattern is found, and that portion of the file is replaced with the short code. New entries are added to the dictionary by suffixing the longest match with the new bits found in the file. Eventually, the whole file is replaced with codes. Decoding reconstructs the same dictionary on-the-fly and looks up the dictionary to convert the codes back to the bit patterns in the files.

3 Overview of Coding Opportunities

We now discuss how we use coding techniques to accelerate ML algorithms. We use the classical Lloyd algorithm for k-means clustering as a detailed running example and then generalize our ideas.

3.1 A Paradigm for Accelerating Machine Learning using Coding

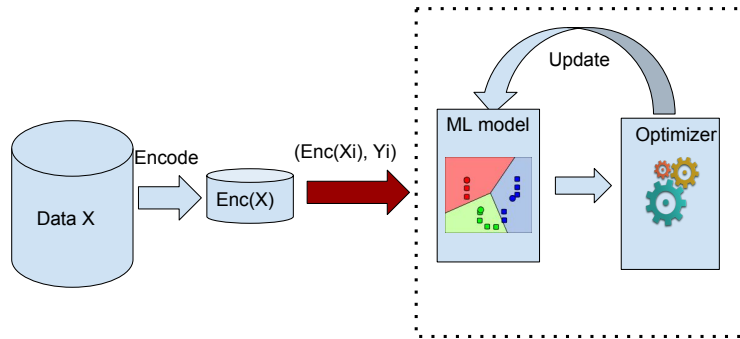


Figure 1: An illustration of our paradigm for using coding to accelerate ML algorithms.

¹www.rarlab.com

²<https://www.pkware.com/pkzip>

³ncompress.sourceforge.net

We first explain our paradigm for accelerating ML algorithms using coding. Common with such accelerations for low-level data processing, our goal is to *encode* the input data and then modify the computations to operate directly over the *encoded data*. Typically, for ML algorithms, the structure of computations is more complicated than that of the low-level data processing operations. For most ML techniques, we have a loss function that is determined by the model structure (for example, the logistic loss for logistic regression), the model parameters, and the input data points. These pieces jointly determine the values of the loss function for data points in the training set. Then, an optimizer updates the model (typically, iteratively) based on the loss until a convergence criterion is met. In the end, the final model parameters are output.

Figure 1 illustrates our paradigm: the codewords, instead of the raw data input, are used as inputs to an ML algorithm along with its model parameters and model structure. Then, an optimization algorithm iteratively optimizes the model. From this paradigm, we can see that a key challenge is that the effectiveness of the encoding in improving runtime performance depends on *both the model structure (most importantly, how the loss function is defined)* and *how the optimization algorithm updates the model during the evaluation steps of the loss function*.

3.2 Running Example: Lloyd’s Algorithm

We use the basic version of Lloyd’s algorithm for k-means to illustrate our ideas. Given a training set S of m data points in a d -dimensional space, this algorithm obtains k clusters under the L_2 -distance metric. Note that Lloyd’s algorithm is iterative. There are three main computations in each iteration. First, given a set of clusters C_1, C_2, \dots, C_k and their corresponding centroids w_1, \dots, w_k , we compute the *loss function* for each x and each w_i :

$$\ell(w_i, x) = \|w_i - x\|_2^2 = \sum_{l=1}^d (w_{i,l} - x_l)^2, \quad (4)$$

Here, $w_{i,l}$ is the l -th component of centroid w_i , and x_l is the l -th component (feature) of x . Second, each data point is assigned to the cluster $C_j : j \in \arg \min_i \ell(w_i, x), i = 1, \dots, k$. This is known as the *assignment step* for x ; we assign each x to its “best current” cluster. Finally, w_i is *updated* as $w_i = \sum_{x \in C_i} x / |C_i|$. Overall, $O(mkd)$ arithmetic operations are performed per iteration.

Opportunity I: Exploiting “Data Redundancy” in the Training Data. We now arrive at our first key observation: *in many datasets, there are often a lot of “data redundancies” among the tuples*. In other words, the feature vectors of two examples, x_i and x_j may share values. Since the computation of $\|w - x_j\|^2$ can be *decomposed* into computations over individual components (Equation (4)), this implies that we can effectively leverage such data redundancy among the tuples in the training set to avoid *redundant computations*. More specifically, given a coding scheme with encoding function enc and decoding function dec , we define the *encoded loss function* as follows:

Definition 3.1 (Encoded Loss Function) *Given a coding scheme, the encoded loss function $\hat{\ell}$, which receives two parameters, a hypothesis w and a codeword $enc(x)$ of x , is defined as $\hat{\ell}(w, enc(x)) = \ell(w, x)$.*

Now, suppose that we use a coding scheme to encode the training set S and that all data points can be encoded using only a small universe of t words, $U = \{c_1, \dots, c_t\}$, where $t \ll m$. In other words, for every j , $enc(x_j)$ can be represented as a tuple $\langle c_{i_1}^j, c_{i_2}^j, \dots, c_{i_s}^j \rangle$: for some $i_1, \dots, i_s \in \{1, \dots, t\}$. Then, $\hat{\ell}(w, enc(x))$ can be computed without decoding. By remembering the squared L_2 -distance between $c_{i_s}^j$ and the corresponding component of w in a *cache table* T , we can compute $\hat{\ell}(w, enc(x))$, which is $\ell(w, x)$, using s queries to T . Note that T is of size kt where we remember for every w_i and every c_s , the squared L_2 -distance between c_s and the corresponding parts in w . Filling T takes at most $O(tkd)$ arithmetic operations. Note that $tkd \ll mkd$ as $t \ll m$.

Since the *average* number of words d' to encode x_j is much smaller than d (i.e. $d' \ll d$), we can compute *all* $\ell(w_i, x_j)$, using $\hat{\ell}(w_i, enc(x_j))$ and T , with a total of at most $O(tkd + mkd')$ arithmetic operations. This is potentially much smaller than $O(mkd)$ as $t \ll m$ and $d' \ll d$.

Opportunity II: Exploiting “Redundancies” in the Codeword Universe. Until now, our discussion applies to *any* source coding scheme. However, we note that for the special class of *prefix source coding*, such as LZW, we can exploit an additional opportunity. Specifically, for prefix codes, the universe $U = \{c_1, \dots, c_t\}$ has the essential property that for $c, c' \in U$, c can be a prefix of c' . Furthermore, this prefix relationship is encoded in the prefix tree where each tree node corresponds to a word $c \in U$, and there is a tree edge from c to c' if c is a prefix of c' . This observation leads to the second opportunity for acceleration: *We can fill in the cache table T faster than filling them separately.* Specifically, once we compute the distance between c and w , we can *reuse* the result to compute the distance between c' and w , instead of redoing the arithmetic operations on the prefix c . As a result, one can fill the table T by *following the topological order, that is level by level, of the prefix tree*, exploiting the prefix redundancy. As a result, one can aim to fill in T with a total number of $N \ll O(tkd)$ arithmetic operations. This gives the final arithmetic complexity $O(N + mkd')$, combining both opportunities, which satisfies that $N + mkd' \ll tkd + mkd' \ll mkd$.

3.3 Generalizations

We now extend the ideas from the last subsection to GLMs.

Unification. We begin by unifying the above two opportunities for the Lloyd’s algorithm. We have two observations: (1) In both cases, the *core task* is to evaluate a certain function f over a dataset $D = \{z_1, \dots, z_n\}$. In the first case, D is the training set S , and in the second case D is the universe of the coding scheme U . (2) Suppose that the coding scheme has encoding function enc and decoding function dec . Observe that what is common to both opportunities is that f is “decomposable” in the sense that for any $z \in D$, if $z = z_1 \circ z_2$ (where \circ is the concatenation operator), then $f(z) = F(f(z_1), f(z_2))$ where F is some combining function. In the Lloyd’s algorithm, for both opportunities, f is the squared L_2 -distance function $f(\cdot) = \ell(w, \cdot)$ (strictly speaking, one should understand this as follows: if $enc(x) = \langle c_1, \dots, c_s \rangle$, then $\ell(w, c_1)$ is defined as the squared 2-norm distance between $dec(c_1)$ and *corresponding components* in w). Note that we can replace the L_2 -distance function by any other common distance function, such as the L_p -distance, and the same argument goes through.

Now, with both (1) and (2), if there is significant data redundancy in D , we can hope to accelerate f on D using our cache table. In the first opportunity for Lloyd’s algorithm, there is redundancy because D is the entire training set. In the second opportunity, D is U and there is redundancy in U as exposed by the prefix tree.

Generalization to GLMs. We note that common optimization algorithms for training GLMs are based on gradient methods, for example *steepest descent*, *conjugate gradient descent*, or *L-BFGS*. For these optimization algorithms, we are in a similar situation as the Lloyd’s algorithm by just replacing the L_2 -distance function by the *inner product* function. As a result, the same two opportunities can be exploited for GLMs with these algorithms as well.

3.4 Discussion

We have shown how to exploit the same opportunities to use coding techniques to accelerate both k-means trained with the Lloyd’s algorithm and GLMs trained with gradient methods. This is intriguing, and somewhat surprising, because k-means and GLMs are superficially quite different ML techniques. We are encouraged by this result, as to the best of our knowledge, this is the first case to expose the opportunities in accelerating multiple ML algorithms using the same coding techniques. Not only do we find that such acceleration is *possible*, but that there are also *common structures* that can be exploited in two fundamental and different ML algorithms.

It is natural then to wonder if our ideas can be extended to more complicated model structures and other optimization algorithms. For example, one could consider kernel methods with appropriate convex optimization algorithms. Even more ambitiously, one could consider deep convolutional neural networks trained with stochastic gradient descent. We are currently exploring these ideas and while we do not have answers to these other models yet, we suspect there could be similar *structures* in those models too that can be exploited using coding techniques. For example, in a convolutional neural network, this might be possible

as the same small kernel is used in all sliding windows in one convolutional layer and the input data also has high redundancy [14, 21, 28]. Overall, we hope our initial ideas and results presented in this paper spur more interest in the community to systematically explore such techniques.

Comparisons with Data Sketching. Our discussion so far may remind astute readers about data sketching techniques such as Gaussian Random Projection [6], where we “compress” the data into lower dimensions, and thus save computation and storage costs. To this end, we have the following points: **(1)** Data sketching techniques are “data independent” (e.g. we apply a random linear transformation A to the data vectors, where A is sampled independently of the data), while our technique is “data dependent.” **(2)** Our technique guarantees that “exactly the same thing is learned as without our technique.” Data sketching techniques, on the other hand, guarantees “approximate” utility with respect to a specific measure, such as distances between points or test accuracy. In that sense, our guarantees of losslessness are stronger. **(3)** Intriguingly, some data sketching techniques can also be applied together with our technique. For example, with a linear transformation A , one may hope that Ax and Ax' have similar redundancies if x and x' have a lot of redundancies.

4 Tuple-Oriented Coding

We now introduce our tuple-oriented coding (TOC) scheme. We first introduce two key differences with the original LZW algorithm, then we explain our coding scheme in detail. Finally we give an example of running our coding scheme on a toy dataset.

4.1 Two Departures from LZW

The first difference is that TOC respects tuple boundaries. Drawing inspiration from LZW, we treat the given training dataset \mathbf{T} as a data file with the data serialized in row-major (tuple-oriented) order. However, as a key departure from LZW and text compression, we also track and maintain tuple boundaries in \mathbf{T} , which are needed for ML. Thus, we introduce a new coding scheme we call tuple-oriented coding (TOC). Using a shared *dictionary*, we encode the data file but prevent the encoding from crossing tuple boundaries.

The second difference is that dictionary entries in TOC are more complicated. Each entry in the dictionary has a *code number* and a 2-tuple: $(startIndex, values)$, wherein *values* is a sequence of feature values that occurs contiguously in some tuple of \mathbf{T} and *startIndex* is the column index of the first feature in *values*. *startIndex* is needed to align *values* with ML model parameters. For example, if the sequence $[c, d]$ of the tuple $[a, b, c, d, e]$ is chosen to be represented as a code, the 2-tuple for it in the dictionary will be $(2, [c, d])$. The code numbers start from 0 and identify each entry uniquely.

4.2 TOC Encoding

TOC starts by initializing with one entry for each unique feature value in \mathbf{T} , i.e., sequences of length one each. The encoding algorithm adds new entries to the dictionary on the fly such that each new sequence is obtained by suffixing one feature value to some pre-existing entry’s sequence. For example, $(2, [c, d])$ is added only if $(2, [c])$ is already present in the dictionary. Apart from being able to access the entry for a given code number, the dictionary also exposes the following two functions:

- $C' = \mathbf{AddDict}(C, v)$. Create a new dictionary entry by extending the sequence of an existing entry with code number C with v and return the new code number. If C is -1, the added entry’s sequence is just v .
- $C' = \mathbf{GetIndex}(C, v)$. Return the code number of the sequence obtained by extending the sequence of an existing entry with code number C with v . If no such extended entry exists, return -1.

The encoding scheme is presented in Algorithm 1. The first step initializes the dictionary with an entry for each starting index (lines 2-3) and then each entry for each unique feature value in \mathbf{T} (lines 4-7). In the second step, for each tuple, for each feature yet to be encoded, we find the longest match W in the dictionary

Algorithm 1: TOC Encoding

Input: Denormalized table \mathbf{T} , Number of features d

```
1 //Step 1: Initialize dictionary
2 for  $i = 0; i < d; i++$  do
3   | AddDict(-1,  $i$ )                                     // Add starting indexes
4 for  $\text{tuple } t \text{ in } \mathbf{T}$  do
5   | for  $i = 0; i < d; i++$  do
6     | | if GetIndex( $i, t[i]$ ) = -1 then
7       | | | AddDict( $i, t[i]$ )                             // Unique feature values
8 //Step 2: Compress  $\mathbf{T}$ 
9 for  $\text{tuple } t \text{ in } \mathbf{T}$  do
10  |  $i = 0$ 
11  | while  $i < d$  do
12  | | ( $C, i$ ) = GetLongestMatch( $i, t, d$ )
13  | | if  $i < d$  then
14  | | | AddDict( $C, t[i]$ )
15  | | Output( $C$ )
```

Algorithm 2: GetLongestMatch

Input: Starting index i , Tuple t , d

```
1  $C' = \text{GetIndex}(i, t[i])$ 
2 while  $C' \neq -1$  do
3   | ( $C, i$ ) = ( $C', i + 1$ )
4   | if  $i < d$  then
5   | |  $C' = \text{GetIndex}(C, t[i])$ 
6   | | else
7   | |  $C' = -1$ 
8 return ( $C, i$ )
```

using the subroutine **GetLongestMatch** shown in Algorithm 2. Once we hit a feature $t[i]$ (i is returned in line 12) that cannot be matched, we add a new entry to the dictionary with the sequence $[W, t[i]]$. We then output the code number corresponding to W to represent the compressed portion of the tuple. This process continues till the whole tuple is encoded. We denote the encoded table \mathbf{T} as \mathbf{T}' .

Example. Figure 2 gives a step-by-step example of how TOC encoding works on a toy dataset with two tuples.

5 Learning on Tuple-Oriented Codes

We now explain how we execute Lloyd/GLM directly on our coded data representation. Since we avoid explicit decoding costs, we call our approach TOC-Based Learning (TOC-Learning for short).

5.1 Basic TOC-Learning

TOC-Lloyd. One iteration of TOC-Lloyd is presented in Algorithm 3. At each iteration, we pre-compute the pair-wise squared distances between the dictionary entries and centroids and store them in the 2-dimensional array \mathbf{B} . Note that the starting index for each dictionary entry is used to align the features of the dictionary entry with the correct feature dimensions in the centroids. The notation $c_{|e.values}$ means that only the dimensions corresponding to the features present in $e.values$ are used. The partial distances are then used

	t[i]	GetLM	AddDict	Output
Step 2 1 st tuple	a	5	12 (0, [a b])	5
	b	6	13 (1, [b, c])	6
	c	7	14 (2, [c, d])	7
	d	8	15 (3, [d, e])	8
	e	9	-	9
Step 2 2 nd tuple	f	10	16 (0, [f, g])	10
	g	11	17 (1, [g, c])	11
	c	14	18 (2, [c, d, e])	14
	e	9	-	9

Figure 2: Illustration of TOC encoding. **T** has two tuples: $[a, b, c, d, e]$ and $[f, g, c, d, e]$. **GetLM** is the output of **GetLongestMatch** for the symbol $t[i]$. **AddDict** shows the entry - $code_number(start_idx, [attrs])$ added to the dictionary upon finding a longest match, where $start_idx$ is the column index of the first attribute. We omit Step 1, which initializes the dictionary with starting indexes: $0(-1, 0)$, $1(-1, 1)$, $2(-1, 2)$, $3(-1, 3)$, and $4(-1, 4)$, as well as single-symbol sequences: $5(0, [a])$, $6(1, [b])$, $7(2, [c])$, $8(3, [d])$, $9(4, [e])$, $10(0, [f])$, and $11(1, [g])$. Subsequent steps add more entries that extend these sequences. As an example, the first row shows that the longest match for a is code number 5, which resulted in the addition of a new entry $(0, [a, b])$ with code number 12. The highlighted row in Step 1 and 2 is where we keep the boundaries of the tuple. That is, when the boundary of a tuple is reached, we stopped matching more characters in **GetLongestMatch** or adding a new entry in **AddDict**. Overall, the compressed tuples are $[5, 6, 7, 8, 9]$ and $[10, 11, 14, 9]$

while computing the full distances between the data points and centroids. The array **DC** tracks how many times each dictionary entry needs to be added (equivalent to adding the examples directly) when computing the new centroids. Thus, TOC-Lloyd might avoid a lot of computational redundancies in both the distance computations and centroid updates.

TOC-GLM. One iteration of TOC-GLM with gradient descent is presented in Algorithm 4. At each iteration, we pre-compute the partial inner products between the dictionary entries and the weights and store them in array **B**. Again, the notation $w_{|e.values}$ means only the dimensions corresponding to the features present in $e.values$ are used. Then, the gradient is computed in one pass over the compressed data, reusing the pre-computed inner-products. We also aggregate the scalar values after applying the scalar function g for each dictionary entry and update the weights by using each dictionary entry once. Recall that $g(a, b)$ is GLM-specific, e.g., for logistic regression it is $-b/(1 + \exp(ab))$, while for linear regression, it is $(a - b)$. Overall, TOC-GLM might avoid a lot of computational redundancy in the inner product computations and weight updates.

5.2 Exploiting Redundancies in Codewords Universe

Efficient Precomputing. We now explain how we efficiently precompute the distances/inner-products between dictionary entries and centroids/weights. These precomputations happen in lines 2-4 in Algorithm 3 and lines 2-3 in Algorithm 4. There are two structures that we leverage to achieve this: the prefix tree structure of the dictionary in TOC, and the structure of the function we evaluate on the dictionary entries.

Algorithm 3: One Iteration of TOC-Lloyd

Input: Compressed \mathbf{T}' , Dict. \mathbf{D} , Centroids \mathbf{C}

```
1 //Initialize new centroids  $\mathbf{C}'$ ,  $\mathbf{B}$ ,  $\mathbf{DC}$ 
2 for each  $e$  in  $\mathbf{D}$  do
3   for each  $c$  in  $\mathbf{C}$  do
4      $\mathbf{B}[e.code, c.CID] = l_2(e.values - c_{|e.values})^2$ 
5   for each  $t$  in  $\mathbf{T}'$  do
6     for each  $c$  in  $\mathbf{C}$  do
7        $s_c = 0$ 
8       for each  $e$  in  $t$  do
9          $s_c += \mathbf{B}[e.code, c.CID]$ 
10       $\hat{c} = \operatorname{argmin}_{c \in \mathbf{C}} s_c$ 
11       $\text{Count}[\hat{c}.CID] += 1$ 
12      for each  $e$  in  $t$  do
13         $\mathbf{DC}[e.code, \hat{c}.CID] += 1$ 
14 for each  $e$  in  $\mathbf{D}$  do
15   for each  $c$  in  $\mathbf{C}'$  do
16      $c_{|e.values} += e.values \times \mathbf{DC}[e.code, c.CID]$ 
17 for each  $c$  in  $\mathbf{C}'$  do
18    $c.x = c.x / \text{Count}[c.CID]$ 
```

Structure I: TOC stores the dictionary entries in a prefix tree. That is, a dictionary entry ($start_idx, [x_0, x_1, \dots, x_n]$) will be stored as a path of nodes starting from root. Note that $start_idx$ will also be stored as a tree node (with special indicating tag).

Structure II: The functions $dist/weighted_sum$ over dictionary entries for Lloyd/GLM respectively are decomposable. Let X be a dictionary entry, X' be its longest prefix, and c be the last attribute,

$$dist(X) = dist(X') + dist(c)$$

$$weighted_sum(X) = weighted_sum(X') + weighted_sum(c)$$

Thus, we evaluate the dictionary entries in the topological/breadth-first traversal order of the prefix tree. Each time we evaluate a tree node, we ask for its parent's evaluated value and add it to the evaluation of the single last attribute, which makes computations faster. Our request for the parent's value is never denied because the structure of the prefix tree makes sure its parent must have been evaluated. The detailed algorithm is shown in Algorithm 5.

Complexity of Our Solution By plugging $dist/weighted_sums$ in $func$ in Algorithm 5, it is easy to compute the complexity of our solution as $O(|D| * k)$ and $O(|D|)$ respectively, where $|D|$ is the dictionary and k is the number of centroids. The naive way to evaluate $dist/weighted_sums$ separately for each dictionary entry has complexity $O(|D| * k * \text{avg. len of dict entries})$ and $O(|D| * \text{avg. len of dict entries})$. Thus, theoretically speaking, our solution achieves a factor of the average length of dictionary entries speedup.

It's also important to note that our solution is cache friendly. In the second step of Algorithm 5, the access to calculated-values is consecutive because adjacent families of children are spawned by adjacent parents in the tree.

Efficient Updating. We introduce an efficient method to update the dictionary entries into the centroids/weight for Lloyd/GLMs. These updates happen at lines 14-16 in Algorithm 3 and lines 11-12 in Algorithm 4.

We discover similar structures as in efficient precomputing. That is, the updating operation using a dictionary entry X is "decomposable" and can be done by updating using the last attribute of X , and then postponing and aggregating the remaining attributes as a whole to the parent of X in the prefix tree.

Algorithm 4: One Iteration of TOC-GLM

Input: Compressed \mathbf{T}' , Dict. \mathbf{D} , Weights w , Labels Y

```
1 //Initialize new gradient  $G$ ,  $\mathbf{B}$ ,  $\mathbf{DW}$ 
2 for each  $e$  in  $\mathbf{D}$  do
3    $\mathbf{B}[e.code] = w'_{e.values} e.values$ 
4 for each  $t$  in  $\mathbf{T}'$  and corresponding  $y$  in  $\mathbf{Y}$  do
5    $z = 0$ 
6   for each  $e$  in  $t$  do
7      $z += \mathbf{B}[e.code]$ 
8    $z = g(z, y)$  //GLM-specific gradient function  $g$ 
9   for each  $e$  in  $t$  do
10     $\mathbf{DW}[e.code] += z$ 
11 for each  $e$  in  $\mathbf{D}$  do
12    $G_{e.values} += e.values \times \mathbf{DW}[e.code]$ 
```

Thus, we update machine learning models using the dictionary entries layer by layer from bottom to top in the prefix tree. For each entry, we only update using the last attribute and postpone/aggregate the remaining part of attributes to its parent. In this process, we only need to update each entry once because the structure of the prefix tree makes sure all the parents of the entry must have been updated beforehand. Algorithm 6 shows the details of our solution.

By plugging updating functions for Lloyd/GLM in **update** in Algorithm 6, the complexity of our updating algorithm is $O(|D| * k)$ and $(|D|)$ for Lloyd/GLM respectively, where $|D|$ is the dictionary and k is the number of centroids. On the other hand, the naive way to update the ML models using each dictionary entry separately has complexity $O(|D| * k * \text{avg. len of dict entries})$ and $O(|D| * \text{avg. len of dict entries})$ for Lloyd and GLM. Thus, again, our method could be the average length of dictionary entries times faster in theory.

6 Experiments

We compare our proposed TOC-learning algorithms with standard learning algorithms using five real-world datasets. Our goal is to answer the following questions:

- *Does the proposed Tuple Oriented Coding (TOC) reduce the data size and thus the storage requirement?*
- *Do the proposed TOC-Learning algorithms provide runtime speedups?*
- *Is there any accuracy loss due to using TOC-Learning algorithms?*
- *Are the proposed TOC-Learning algorithms parallelizable?*
- *How large are the end to end performance benefits achieved by TOC-Learning algorithms compared to state-of-the-art machine learning toolkits?*

We answer those questions positively by conducting extensive experiments on several real-world datasets. First, we notice that TOC does reduce the data sizes notably, with compression ratios up to 14x. Our proposed TOC-Learning algorithms also achieve significant runtime speedups, up to 14x. Using a lossless compression encoding, LZW, TOC-Learning algorithms obtain exactly the same output as the ordinary learning algorithms (assuming exact arithmetic), which is verified by the experiments. We also show the parallelizability of TOC-Learning algorithms by running them on multi-core systems. Finally, TOC-Learning algorithms demonstrate a significantly better performance compared to Scikit-learn, a well-tuned machine learning toolkit whose kernel is written in C. In fact, the speedups can be as high as 31.2x. In the remainder of this section, we will discuss the above results in details.

Algorithm 5: Efficiently Precompute Intermediate Results on Dictionary Entries

```
1 // Step1: This is pre-computing. Once per ML
  Input: The root of the prefix tree during the compression stage of LZW
2 tree-node-list = [root]
3 dic-entry-array = []
4 idx = 0
5 while tree-node-list not empty do
6   node = tree-node-list.pop_front()
7   if node has been outputed as a code then
8     dict-entry-array[idx] = node
9     idx += 1
10  for child of node do
11    child.set_prefix_idx(idx)
12    tree-node-list.push_back(child)
13 // Step2: This is invoked basically once per iteration
14 calculated-values=[]
15 for i = 0 to len(dic-entry-array) do
16   if dic-entry-array[i] has prefix_idx then
17     calculated-values[i] = calculated-values[prefix_idx] + func(last attr value of dic-entry-array[i])
18   else
19     calculated-values[i] = func(last attr value of dic-entry-array[i])
```

Algorithm 6: Efficiently Update Machine Learning Models Using Dictionary Entries

```
1 // Step1: We reuse the dic-entry-array in Algorithm 5.
2 // Step2: This is invoked basically once per iteration
3 update-scalar-vec // this should be pre-poluated by ML models
4 for i = dic-entry-array.end to head do
5   if dic-entry-array[i] has prefix_idx then
6     update-scalar-vec[prefix_idx] += update-scalar-vec[i]
7   update(last attr value of dic-entry-array[i], update-scalar-vec[i])
```

Experimental Setup. All experiments were run on CloudLab [30] using machines with Intel Xeon 2.2 GHz CPU, 256GB RAM, and Ubuntu 14.04.1. Our code is written in C++ and compiled using g++ 4.8.4 with the “O3” flag.

Datasets. We used five real-world datasets obtained from [25].⁴ The dimensions of the tables are listed in Table 1. For details about these datasets’ schemas and features, we refer the interested reader to [25].

Notation. For k-means, let Lloyd and TOC-Lloyd denote the original Lloyd’s algorithm on the raw data and our proposed TOC-Learning Lloyd algorithm on the encoded data, respectively. For Generalized Linear Models, similarly let BGD and TOC-BGD be the original BGD algorithm on the raw data and our proposed TOC-Learning BGD algorithm on the encoded data, respectively. Note that more advanced optimization methods such as L-BFGS and Conjugate Gradient have the same data access pattern as BGD. Thus our coding techniques can be easily applied to them too.

⁴We convert the categorical features in these datasets using one-hot encoding. We also encode the converted dataset in the sparse format to avoid creating large sparse features.

Dataset	data size	attribute type
Covtype	581012 * 54	Categorical, Integer
US Census	2458285 * 68	Categorical
Dota2	102944 * 116	Integer, real
Kdd99	4000000 * 42	Categorical, Integer
Facebook	40949 * 54	Integer, Real

Table 1: Real dataset statistics.

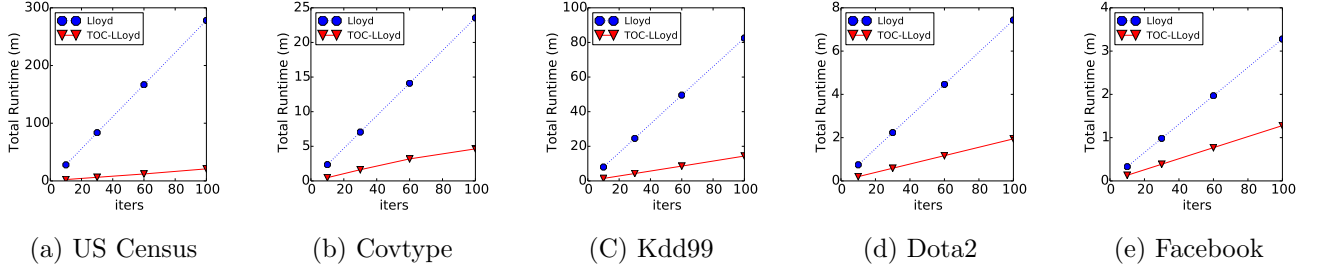


Figure 3: The k-means runtimes on (a) Census, (b) Covtype, (c) Kdd99, (d) Dota2, and (e) Facebook. We vary the number of iterations.

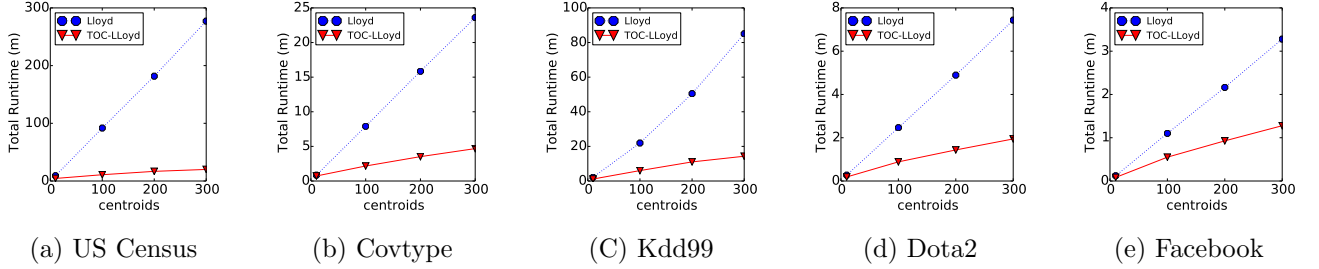


Figure 4: The k-means runtimes on (a) Census, (b) Covtype, (c) Kdd99, (d) Dota2, and (e) Facebook. We vary the number of centroids.

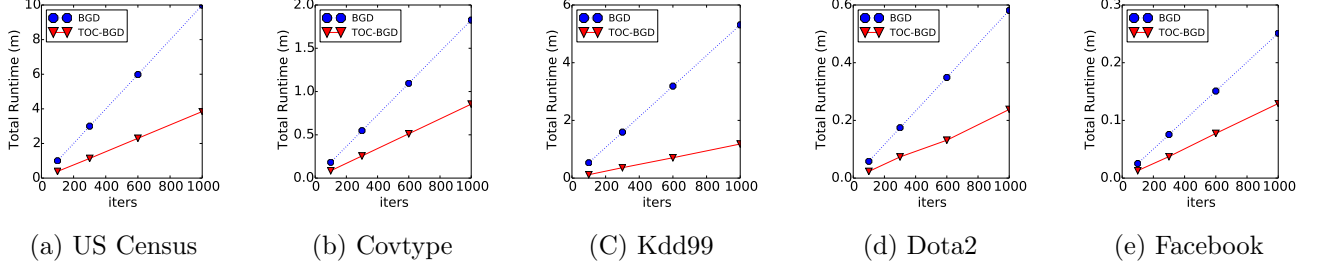


Figure 5: The logistic regression runtimes on (a) Census, (b) Covtype, (c) Kdd99, (d) Dota2, and (e) Facebook. We vary the number of iterations.

6.1 Storage Reduction

Table 2 shows the compression ratios of proposed TOC encoding on five real-world datasets. We see that TOC encoding can significantly reduce the dataset sizes. Note that the compression ratios are slightly smaller than those of using gzip, a well-tuned compression software. This is because we have not fully tuned our encoding scheme for storage reduction. It is interesting future work to further optimize our encoding schemes.

Dataset	Original Size	TOC	gzip
Covtype	72M	3.5x	6.6x
US Census	1651M	14.2x	16.0x
Dota2	23M	3.1x	11.5x
Kdd99	1758M	37.1x	65.1x
Facebook	26M	5.0x	13.0x

Table 2: Compression Ratios on Real Datasets

6.2 Runtime Speedup

Now we dive deep into the runtime speedups of our proposed TOC-Learning algorithms.

Lloyd. Figures 3 and 4 present the runtime performance comparison between our proposed TOC-Lloyd and the standard Lloyd’s algorithm. In Figure 3, the number of centroids is fixed to be 300; in Figure 4, the number of iterations is fixed to be 100. Note that only the runtimes of the learning algorithms are reported, since encoding data is a one-time cost.

As shown in Figure 3, our TOC-Lloyd is consistently faster than the standard Lloyd’s algorithm for all five datasets no matter how many iterations there are. The speedups are up to 13.5x. This validates the effectiveness of our encoding techniques. Note that the runtimes are linear with respect to the number of iterations.

Figure 4 illustrates that the higher the number of centroids, the larger the speedup. This is because as the number of centroids increases, the number of precomputed distances between a dictionary entry and the centroids that are stored in the dictionary also increases. Therefore, each table lookup corresponds to a larger block of memory and the unit cost of random memory access is reduced.

General Linear Models. Now we demonstrate the runtime speedup of TOC-Learning batch gradient descent (TOC-BGD) against standard BGD on training GLM models, using logistic regression as an example. Figure 5 shows that our TOC-BGD is consistently faster than its counterpart. Interestingly, the speedups are smaller than those of TOC-Lloyd with a large number of centroids. This is primarily because each table lookup of TOC-BGD only obtains the inner product of a code and its corresponding part in a weight vector, while each table lookup of TOC-Lloyd obtains the inner product of a code and its corresponding parts in all centroids, whose size is linear with respect to the number of centroids. In fact, when the number of centroids is small, the speedup of TOC-Lloyd is similar to or smaller than that of TOC-BGD.

6.3 Parallelism

We parallelized our TOC-Lloyd using OpenMP⁵ and ran our algorithm in a shared-memory multi-core machine by varying the number of cores. Figure 6 shows that this can notably reduce the runtime of TOC-Lloyd by 75% using 10 cores.

6.4 End to End Performance Gains Compared with Scikit-learn

Finally, we illustrate the end to end performance gains our compressed Lloyd can achieve compared to the Lloyd implementation in Scikit-learn, a popular machine learning toolkit with a highly optimized kernel written in c. The number of iterations and centroids are set to be 100 and 300, respectively. As shown in Table 3, the speedups can be up to 31.2x. This shows that integrating our encoding techniques with state-of-the-art machine learning libraries may provide appealing end-to-end performance gains.

Table 3 also reports the encoding time of all the five datasets. Overall, they are smaller compared to the algorithm running time.

⁵www.openmp.org

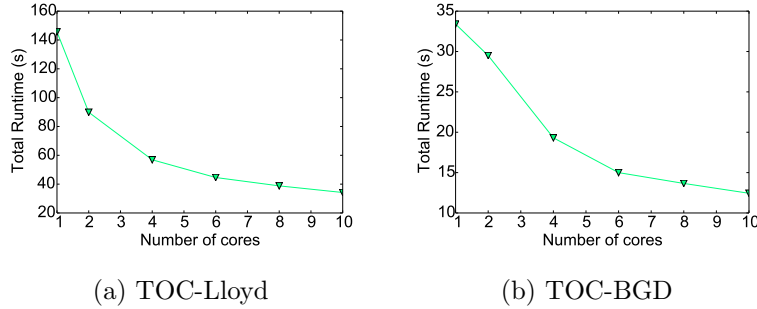


Figure 6: Parallelism using a shared-memory multi-cores machine for (a) TOC-Lloyd (b) TOC-BGD

Dataset	Coding Time	TOC-Lloyd	Sk Lloyd	SpeedUp
Covtype	5.0s	277.8s	8680.2s	31.2x
US Census	24.4s	1253.9s	*	*
Dota2	2.3s	116.5s	1213.4s	10.5x
Kdd99	7.4s	862.2s	13358.s	15.5x
Facebook	0.8s	76.7s	152.2s	2.0x

Table 3: Runtimes comparison between our compressed Lloyd with Lloyd in Scikit-learn (Sk), a well-tuned machine learning toolkit. * means the experiments do not terminate after 24 hours.

7 Related Work

There have been two independent research lines that explore: (1) using coding techniques, as what we do in this paper, to accelerate *low-level data processing*, and (2) exploiting *redundancy exposed by the relational database schema* to accelerate machine learning. Our work differs from both in that we apply coding techniques to accelerate machine learning. Compared to the first direction, the computation we accelerate is more advanced and complicated. Compared to the second direction, our work does not need the existence of a schema to work with. Moreover, we can “automatically” identify data redundancy, and further computation redundancy, to accelerate machine learning. As a result, we do not require “human effort” to manually define a database schema to expose the redundancy.

Compressed Query Processing and Analytics. RDBMSs have used data compression to reduce storage requirements and improve query efficiency [1, 8, 18, 35]. More recently, column-store databases use columnar compression with SQL query processing [1], providing high performance for SQL analytics. Our work is inspired by these works though we are not aware of previous works that use coding techniques to accelerate *machine learning*. As another line, recent work on compressed neural networks [7, 16] shows that compression can be used to compress the model trained by neural networks. However, these techniques are not designed to speed up the training process. Recent work on compressed linear algebra (CLA) shows that columnar compression for matrices (tables) could help reduce file sizes for linear algebra-based ML programs in SystemML [11]. However, CLA mainly focuses on file size reductions to fit big datasets into memory. In contrast, our proposed coding schemes aim for reducing CPU computations while obtaining a smaller encoded dataset as a byproduct. It is interesting future work to expand our encoding schemes to these linear algebra operations.

Factorized Machine Learning and Factorized Databases. Factorized machine learning techniques [22, 31] push machine learning computations through joins and avoid the schema-based redundancy on denormalized datasets, and factorized databases do the same for relational operations [4, 5]. While these works and ours share the same goal of accelerating machine learning, our work also differs significantly in several aspects. First, we do not need a schema to start with, and can directly work on a single flat table and accelerate the training. Note that most datasets for machine learning are available in the format of a single

flat table. Second, our work suggests that coding techniques can be used to “automatically extract” redundancies in a dataset. In contrast, factorized machine learning requires a manually defined schema to expose redundancy, and need to manage complex multi-table schemas or database dependencies. It is, however, interesting to compare these two approaches on the performance improvement.

8 Conclusion and Future Work

Providing efficient machine learning algorithms has become crucial for many applications. In this paper, we study the problem of accelerating machine learning workloads using encoding techniques. As a first step towards this direction, we show, perhaps surprisingly, that the classic LZW compression method can be used to speed up k-means using Lloyd’s algorithm and Generalized Linear Models using batch gradient descent. Experimental results show that our proposed TOC learning algorithms can achieve significant speedup compared to Scikit-learn, a well-tuned machine learning toolkit. It is interesting future work to explore how to speed up other learning models, such as deep neural networks, and other learning algorithms, such as stochastic gradient descent. Another avenue is to investigate scalable TOC learning algorithms in distributed systems.

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